On interpretation of the spin structure functions

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Abstract

The spin structure functions of the system of quasifree fermions on mass shell are studied in a consistently covariant approach. Comparison with the basic formulas following from the quark-parton model reveals the importance of the fermion motion inside the target for the correct evaluation of the spin structure functions. In particular it is shown, that regarding the moment Γ_1, both the approaches are equivalent for the static fermions, but differ by the factor $1/3$ in the limit of massles fermions ($m \ll p_0$, in target rest frame). Some other summation rules are discussed as well.

1 Introduction

Measuring of the nucleon spin structure functions represents an important tool not only for better understanding of the nucleon internal structure in the language of the QCD, but also for better understanding of QCD itself. These functions contain an information, which is a crucial complement to the structure functions obtained in the unpolarized deep inelastic scattering (DIS) experiments.

The polarized experiments are more complex and difficult than the unpolarized ones, nevertheless the last decade has brought remarkable results also for the nucleon spin functions from the experiments at CERN (EMC, SMC) and SLAC (E142, E143, E154, E155). And the new experiments are running (HERMES) or are being under preparation (COMPASS). The data on polarized pp collisions are expected from the collider RHIC. For the present status of the research in structure functions see e.g. [1], the overview [2] and citation therein. The more formal aspects of the polarized DIS are explained in [3].

Also the interpretation and understanding of polarized structure functions seem be more difficult. For an example, until now it is not well understood, why the integral of the proton spin structure function g_1 is substantially less, than expected from very natural assumption, that the nucleon spin is generated by the valence quarks. Presently, there is a tendency to explain the missing part of the nucleon spin as a contribution of the gluons. It has been also suggested, that the quark orbital momentum can play some role as well [4]-[6].

The spin in general is a very delicate quantity, which requires correspondingly precise treatment. It has been argued, that for correct evaluation the quark contribution to the nucleon spin it is necessary to take properly into account the internal quark motion [4] - [13]. Necessity of the covariant formulation of the quark - parton model (QPM) for the spin functions has been pointed out in [14]. These requirements are not satisfied in the standard formulation of the QPM, which is currently used for analysis and interpretation of the experimental data.

In this paper we shall attempt to demonstrate the role of the internal motion for the spin structure functions, using very simple model of the system quasifree fermions on mass shell. The basic requirement is consistently covariant formulation of the task for the system of fermions, which are not static, being characterized by some momenta distribution in the frame of their centre of mass. The spin structure functions of such system are obtained in Sec. 2 and the summation rules following from these functions are shown in Sec. 3. In the Sec. 4 a comparison with the formulas of the standard QPM is done. The last section is devoted to the short summary.

2 Spin structure functions in covariant approach

Let us imagine a system of three quasifree charged fermions with the spin 1/2 and mass m, for which the following conditions are satisfied:

1) The distribution of their momenta in given reference frame is described by some spherically symmetric function G,

$$\int G(p_0)d^3p = 3; \qquad p_0 = \sqrt{m^2 + \vec{p}^2}. \tag{1}$$

The free fermion states are described by the spinors

$$\psi_{p,\lambda}(x) = \frac{1}{\sqrt{\Omega}} u\,(p,\lambda) \exp(ipx); \qquad \int_\Omega \psi^\dagger_{p,\lambda}(x)\psi_{p,\lambda}(x)d^3x = 1, \tag{2}$$

where Ω is the normalization volume and

$$u\,(p,\lambda) = \frac{1}{\sqrt{N}} \begin{pmatrix} \phi_\lambda \\ \frac{\vec{p}\vec{\sigma}}{p_0+m}\phi_\lambda \end{pmatrix}; \qquad N = \frac{2p_0}{p_0+m}, \qquad \phi^\dagger_\lambda \phi_\lambda = 1. \tag{3}$$

We assume

$$\frac{1}{2}\vec{S}\vec{\sigma}\phi_\lambda = \lambda\phi_\lambda, \qquad \lambda = \pm\frac{1}{2}, \tag{4}$$

which means, that the spin projection of the fermion in its rest frame is $\pm 1/2$ in the given direction $\vec{S}$; $\left|\vec{S}\right| = 1$.

2) By $G^{\pm}$ we denote function, which measures probability, that fermion is in the state $\psi_{p,\pm 1/2}$, so that

$$G(p_0) = G^+(p_0) + G^-(p_0) \tag{5}$$

and we assume

$$\int \Delta G(p_0) d^3p = 1; \qquad \Delta G(p_0) \equiv G^+(p_0) - G^-(p_0). \tag{6}$$

The difference ΔG consists of the corresponding contributions Δh_j from the three fermions:

$$\Delta G(p_0) = \sum_{j=1}^{3} \Delta h_j(p_0); \qquad \Delta h_j(p_0) \equiv h^+(p_0) - h^-(p_0). \tag{7}$$

Later on, we shall need also the distribution

$$H(p_0) \equiv \sum_{j=1}^{3} e_j^2 \Delta h_j(p_0), \tag{8}$$

where e_j are the fermion charges.

What is the resulting spin (total angular momentum) and its projection related to the whole system? Apparently, one needs to calculate the integral of the matrix elements

$$\left\langle \vec{S}\vec{j} \right\rangle = \int \int_{\Omega} G(p_0) \left(\psi^{\dagger}_{p,\lambda}(x) \vec{S}\vec{j} \psi_{p,\lambda}(x) \right) d^3x d^3p, \tag{9}$$

where the angular momentum $\vec{j}$ consists of the spin and orbital part

$$j_k = \Sigma_k + l_k = \frac{1}{2} \begin{pmatrix} \sigma_k & 0 \\ 0 & \sigma_k \end{pmatrix} - i\varepsilon_{klm} p_l \frac{\partial}{\partial p_m}. \tag{10}$$

Since the total angular momentum $\vec{j}$ is a conserving quantity, which commutes with the term $\vec{p}\vec{\sigma}$, a simple calculation gives

$$\psi^{\dagger}_{p,\lambda}(x) \vec{S}\vec{j} \psi_{p,\lambda}(x) = \frac{1}{\Omega} \left(\lambda - \varepsilon_{klm} S_k p_l x_m \right). \tag{11}$$

So, after inserting to Eq. (9) and using the assumption (6) one gets

$$\left\langle \vec{S}\vec{j} \right\rangle = \frac{1}{\Omega} \int \int_{\Omega} \left[\left(G^+(p_0) - G^-(p_0) \right) / 2 - G(p_0) \varepsilon_{klm} S_k p_l x_m \right] d^3x d^3p \tag{12}$$

$$= \frac{1}{2} \int \Delta G(p_0) d^3p = \frac{1}{2},$$

since the term $\varepsilon_{klm} S_k p_l x_m$, due to spheric symmetry, vanishes. This means, that the whole system is in the state with spin 1/2 and its projection 1/2 with respect to the axis of quantization $\vec{S}$, in the other words, the system is polarized in this direction. Let us point out, in the relativistic case, having state with definite projection $\vec{S}\vec{j}$ of the total angular momentum, one cannot separate its orbital and spin part (with exception of the special case when $\vec{S} \parallel \vec{p}$), i.e. account with the fermion orbital momentum is crucial for a consistent calculation of the resulting spin. On the other hand, the similar calculation, in which the orbital part $\vec{l}$ is ignored, gives

$$\psi_{p,\lambda}^{\dagger}(x)\vec{S}\vec{\Sigma}\psi_{p,\lambda}(x) = \frac{1}{\Omega N}\left(\lambda\phi_{\lambda}^{\dagger}\phi_{\lambda} + \phi_{\lambda}^{\dagger}\frac{\vec{p}\vec{\sigma}\cdot\vec{S}\vec{\sigma}\cdot\vec{p}\vec{\sigma}}{2\left(p_0+m\right)^2}\phi_{\lambda}\right)$$

$$= \frac{1}{\Omega N}\left(\lambda + \phi_{\lambda}^{\dagger}\frac{\vec{p}\vec{\sigma}\cdot\left(-\vec{p}\vec{\sigma}\cdot\vec{S}\vec{\sigma} + 2\vec{p}\vec{S}\right)}{2\left(p_0+m\right)^2}\phi_{\lambda}\right)$$

$$= \frac{1}{\Omega N}\left(\lambda - \lambda\frac{\left|\vec{p}\right|^2}{\left(p_0+m\right)^2} + \phi_{\lambda}^{\dagger}\frac{\vec{p}\vec{\sigma}\cdot\vec{p}\vec{S}}{\left(p_0+m\right)^2}\phi_{\lambda}\right).$$

Since

$$\vec{p}\vec{\sigma}\cdot\vec{p}\vec{S} = \sum_{i=1}^{3} p_i^2\sigma_i S_i + \sum_{j\neq i} p_i p_j \sigma_i S_j \tag{13}$$

one can write

$$\left\langle \vec{S}\vec{\Sigma}\right\rangle = \int\int_{\Omega} G(p_0)\left(\psi_{p,\lambda}^{\dagger}(x)\vec{S}\vec{\Sigma}\psi_{p,\lambda}(x)\right)d^3x d^3p$$

$$= \int G(p_0)\frac{\lambda}{N}\left(1 - \frac{\left|\vec{p}\right|^2}{\left(p_0+m\right)^2} + \frac{2\left|\vec{p}\right|^2}{3\left(p_0+m\right)^2}\right)d^3p,$$

where inserting the formula (13), we take into account, that due to spheric symmetry the terms $p_i p_j$ $(j \neq i)$ vanish and the terms p_i^2 can be substituted by $\left|\vec{p}\right|^2/3$. The last relation can be further simplified:

$$\left\langle \vec{S}\vec{\Sigma}\right\rangle = \frac{1}{2}\int \Delta G(p_0)\left(\frac{1}{3} + \frac{2m}{3p_0}\right)d^3p \leq \frac{1}{2}. \tag{14}$$

One can observe, that the correspondence with Eq. (12) takes place only for the system of *static* fermions.

For further consideration, it will be useful to substitute the vector $\vec{S}$, representing the direction of the fermion polarization, by the corresponding covariant polarization vector w^σ, which satisfies

$$w^2 = -1, \qquad wp = 0 \tag{15}$$

and $w = (0, \vec{S})$ in the fermion rest frame. The explicit representation of the vector w will be defined hereinafter.

Now, let us expose this system as a (fixed) target to the beam of polarized electrons (e.g. $helicity = 1/2$) coming with the momentum

$$k = \left(k_0, \sqrt{k_0^2 - m_e^2}, 0, 0\right) \tag{16}$$

and let us calculate the form of corresponding differential cross-section. The spin dependent part of the cross-section for interaction with a single fermion in one photon approximation has the form

$$d\sigma \sim -L^{\alpha\beta(A)}(q,s)T_{\alpha\beta}^{(A)}. \tag{17}$$

The antisymmetric tensor $L^{\alpha\beta(A)}$, (see e.g. [3]) related to the electron beam reads:

$$L^{\alpha\beta(A)} = m_e \varepsilon_{\alpha\beta\lambda\sigma} s^\lambda q^\sigma, \tag{18}$$

where m_e is the electron mass, s denotes its polarization vector

$$s = \frac{1}{m_e}\left(\sqrt{k_0^2 - m_e^2}, k_0, 0, 0\right); \qquad s^2 = -1, \qquad ks = 0 \tag{19}$$

and $q = k - k'$ is the photon momentum. The antisymmetric tensor $T^{\alpha\beta(A)}$ related to the single fermion inside the target has a similar form:

$$T^{\alpha\beta(A)} = m\varepsilon_{\alpha\beta\lambda\sigma} q^\lambda w^\sigma, \tag{20}$$

where m and w denote the fermion mass and polarization vector. If one assumes, that the electron scattering can be described as the incoherent sum of the interactions with three the fermions, than the tensor $T^{\alpha\beta(A)}$ reads

$$T_{\alpha\beta}^{(A)} = \varepsilon_{\alpha\beta\lambda\sigma} q^\lambda m \int H(p_0) w^\sigma \delta((p+q)^2 - m^2)\frac{d^3p}{p_0}. \tag{21}$$

Here the charge factors are included into the tensor through the distribution (8). Further, we can modify the $\delta-$function term:

$$\delta((p+q)^2 - m^2)d^3p = \delta(2pq + q^2)d^3p = \frac{1}{2\xi}\delta(\frac{pq}{\xi} + \frac{q^2}{2\xi})d^3p, \tag{22}$$

where ξ is arbitrary constant, which only rescales the integration variable. Now, let us imagine, that our target is a part of the greater system, which is at rest with respect to the given reference frame and has the mass M, but at the same time the probing electron interact only with three the fermions. If we put

$$\xi = Mq_0 = M\nu, \tag{23}$$

then in the $\delta-$function one can identify the terms known from the formalism of deep inelastic scattering:

$$-\frac{q^2}{2M\nu} = \frac{Q^2}{2M\nu} = x \tag{24}$$

which is the Bjorken scaling variable, its value can be directly determined using only initial and final momenta of the scattered electron. This variable is in the $\delta-$function compensated by the ratio $pq/M\nu$, which after boosting the whole target of mass M to the infinite momentum frame approximately represents ratio of dominating momenta components p'/P' of the fermion and the target.

The explicit form of the polarization vector w can be found as follows. First, let us transform the vector $w = (0, \vec{S})$ from the fermion rest frame to the target rest frame. After decomposition of the vector $\vec{S}$ to longitudinal and transversal parts with respect to the momentum fermion $\vec{p}$, the corresponding Lorentz boost gives

$$(0, \vec{S}) \to w = \left(\frac{\vec{p}\vec{S}}{m}, \vec{S} + \frac{\vec{p}\vec{S}}{m(m+p_0)}\vec{p} \right). \tag{25}$$

Secondly, let us make a Lorentz boost of the whole target with mass M to some another frame, which is defined by the new components of the target momentum

$$(M, 0, 0, 0) \to P = \left(P_0, \vec{P}\right); \qquad P^2 = M^2. \tag{26}$$

Next, if we define the covariant vector S by its components in the target rest frame as

$$S = (0, \vec{S}), \tag{27}$$

then the polarization vector w can be written in manifestly covariant form

$$w^\sigma = AP^\sigma + BS^\sigma + Cp^\sigma, \tag{28}$$

where A, B, C are invariant functions (scalars) of the vectors P, S, p. These three functions are fixed by two the conditions (15) and by the constraint (25) valid in the target rest frame. A simple calculation gives:

$$A = -\frac{pS}{pP + mM}, \qquad B = 1, \qquad C = \frac{M}{m}A. \tag{29}$$

So, we have obtained explicit covariant form of the polarization vector w entering the tensor (21), which can be now in accordance with the relations (22)-(24) rewritten

$$T^{(A)}_{\alpha\beta} = \varepsilon_{\alpha\beta\lambda\sigma} q^\lambda \frac{m}{2Pq} \int H\left(\frac{pP}{M}\right) w^\sigma \delta\left(\frac{pq}{Pq} - x\right) \frac{d^3p}{p_0}, \tag{30}$$

where we use the invariant term Pq instead of $M\nu$ and $H(pP/M)$ instead of $H(p_0)$.

On the other hand, in accordance with the general rule (see e.g. [3]), the antisymmetric tensor $T^{(A)}_{\alpha\beta}$ appearing in the formula for the cross-section (17), has the form

$$T^{(A)}_{\alpha\beta} = \varepsilon_{\alpha\beta\lambda\sigma} q^{\lambda} \left\{ M S^{\sigma} G_1 + [(Pq)S^{\sigma} - (qS)P^{\sigma}] \frac{G_2}{M} \right\}, \tag{31}$$

where M, P, S represent the target mass, momentum and spin polarization vector, which satisfies

$$S^2 = -1, \qquad PS = 0. \tag{32}$$

The invariants G_1 and G_2 are the spin structure functions. In the next we shall identify the parameters M, P, S in Eq. (31) with those in the model described above and simultaneously we shall attempt to determine the spin structure functions corresponding to our target. First of all, we modify the Eq. (31) by the substitution

$$G_S = M G_1 + \frac{Pq}{M} G_2, \quad G_P = \frac{qS}{M} G_2, \tag{33}$$

which gives

$$T^{(A)}_{\alpha\beta} = \varepsilon_{\alpha\beta\lambda\sigma} q^{\lambda} \left\{ S^{\sigma} G_S - P^{\sigma} G_P \right\}. \tag{34}$$

Comparison with Eq. (30) gives the equation for the structure functions:

$$\varepsilon_{\alpha\beta\lambda\sigma} q^{\lambda} \left\{ S^{\sigma} G_S - P^{\sigma} G_P \right\} = \varepsilon_{\alpha\beta\lambda\sigma} q^{\lambda} \frac{m}{2Pq} \int H\left(\frac{pP}{M}\right) w^{\sigma} \delta\left(\frac{pq}{Pq} - x\right) \frac{d^3 p}{p_0}. \tag{35}$$

How one can solve such equation for G_S, G_P? Generally, having equation like

$$\varepsilon_{\alpha\beta\lambda\sigma} q^{\lambda} y^{\sigma} = \varepsilon_{\alpha\beta\lambda\sigma} q^{\lambda} z^{\sigma}, \tag{36}$$

one can contract it first by $\varepsilon^{\alpha\beta\mu\nu}$, which gives

$$q^{\mu} y^{\nu} - q^{\nu} y^{\mu} = q^{\mu} z^{\nu} - q^{\nu} z^{\mu}$$

and then by q_{μ}, which implies

$$y^{\nu} - z^{\nu} = q^{\nu} \frac{q(y-z)}{q^2}. \tag{37}$$

This means, that Eq. (36) is satisfied for any two vectors y, z for which

$$y^{\nu} - z^{\nu} = D q^{\nu}, \tag{38}$$

where D is arbitrary scalar. In this way the Eq. (35) combined with the relation (28) imply

$$S^\sigma G_S - P^\sigma G_P = \frac{m}{2Pq}\int H\left(\frac{pP}{M}\right)\left(AP^\sigma + BS^\sigma + Cp^\sigma\right)\delta\left(\frac{pq}{Pq} - x\right)\frac{d^3p}{p_0} + Dq^\sigma, \tag{39}$$

where the functions A, B, C are given by relations (29) and G_S, G_P and D are unknown. After contracting with P_σ, S_σ and q_σ one gets the equations

$$-M^2 G_P = \frac{m}{2Pq}\int H\left(\frac{pP}{M}\right)\left(AM^2 + C\cdot pP\right)\delta\left(\frac{pq}{Pq} - x\right)\frac{d^3p}{p_0} + D\cdot Pq, \tag{40}$$

$$-G_S = \frac{m}{2Pq}\int H\left(\frac{pP}{M}\right)\left(-B + C\cdot pS\right)\delta\left(\frac{pq}{Pq} - x\right)\frac{d^3p}{p_0} + D\cdot qS, \tag{41}$$

$$qS\cdot G_S - Pq\cdot G_P = \frac{m}{2Pq}\int H\left(\frac{pP}{M}\right)\left(A\cdot Pq + B\cdot qS + C\cdot pq\right) \tag{42}$$

$$\times\delta\left(\frac{pq}{Pq} - x\right)\frac{d^3p}{p_0} + Dq^2.$$

and inserting G_P, G_S from the first two equations to the last one gives the condition for D:

$$\frac{m}{2Pq}\int H\left(\frac{pP}{M}\right)\left(C\cdot pu\right)\delta\left(\frac{pq}{Pq} - x\right)\frac{d^3p}{p_0} + D\cdot qu = 0, \tag{43}$$

where we denote

$$u \equiv q + (qS)\,S - \frac{(Pq)}{M^2}P.$$

Finally, inserting D from this equation to Eqs. (40), (41) gives with the use of relations (29) the structure functions

$$G_P = \frac{m}{2Pq}\int H\left(\frac{pP}{M}\right)\frac{pS}{pP + mM}\left[1 + \frac{1}{mM}\left(pP - \frac{pu}{qu}Pq\right)\right]\delta\left(\frac{pq}{Pq} - x\right)\frac{d^3p}{p_0}, \tag{44}$$

$$G_S = \frac{m}{2Pq}\int H\left(\frac{pP}{M}\right)\left[1 + \frac{pS}{pP + mM}\frac{M}{m}\left(pS - \frac{pu}{qu}qS\right)\right]\delta\left(\frac{pq}{Pq} - x\right)\frac{d^3p}{p_0}. \tag{45}$$

The spin structure functions in the standard notation $g_1 = M\cdot Pq\cdot G_1$, $g_2 = (Pq)^2/M\cdot G_2$ can be now obtained from Eqs. (33):

$$g_1 = Pq\left(G_S - \frac{Pq}{qS}G_P\right), \qquad g_2 = \frac{(Pq)^2}{qS}G_P, \qquad g_1 + g_2 = PqG_S, \tag{46}$$

where the functions G_S, G_P are given by relations (44), (45). Corresponding integrals, as shown in the Appendix, can be simplified to the form (83), (84).

3 Summation rules

For next analysis of the obtained structure functions it is convenient to express the integrals (44),(45) in the target rest frame, where $P = (M, 0, 0, 0)$ and $S = (0, \vec{S})$. Detailed calculation is done in the Appendix. Using Eq. (76) one gets

$$\Gamma_P \equiv \int G_P dx = \frac{\pi \cos\omega}{M^2\nu} \int H(p_0) \left(p_1 + \frac{\nu}{|\vec{q}|} \frac{p_1^2 - p_T^2/2}{p_0 + m} \right) \frac{p_T dp_1 dp_T}{p_0} \tag{47}$$

and due to spheric symmetry of the distribution H, the terms proportional to p_1 and $p_1^2 - p_T^2/2$ vanish, insofar that

$$\Gamma_P = 0. \tag{48}$$

This result, with the use of the second relation (46), implies

$$\Gamma_2 \equiv \int g_2 dx = 0, \tag{49}$$

which is the known Burkhardt-Cottingham sum rule [15]. Similarly the Eq. (77) gives

$$\Gamma_S \equiv \int G_S dx = \frac{\pi}{M\nu} \int H(p_0) \left(m + \frac{p_T^2}{2\,(p_0 + m)} \right) \frac{p_T dp_1 dp_T}{p_0}. \tag{50}$$

After the substitution

$$2\pi p_T dp_1 dp_T = d^3p \tag{51}$$

one gets

$$\Gamma_S = \frac{1}{2M\nu} \int H(p_0) \left(m + \frac{\vec{p}^2}{3\,(p_0 + m)} \right) \frac{d^3p}{p_0}, \tag{52}$$

$$= \frac{1}{2M\nu} \int H(p_0) \left(\frac{1}{3} + \frac{2m}{3p_0} \right) d^3p$$

which with the use of the first relation (46) and (48) gives

$$\Gamma_1 \equiv \int g_1 dx = \frac{1}{2} \int H(p_0) \left(\frac{1}{3} + \frac{2m}{3p_0} \right) d^3p. \tag{53}$$

This result can be compared with Eq. (14). Both the functions H and ΔG have the defined normalizations and the corresponding integrals are equal to these normalized values in the limit, when the fermions are static ($p_0 = m$). On the other hand in the limit of massless fermions ($m \ll p_0$) these integrals represent only one third of their normalized value. Apparently the Γ_1 "measures" only the mean contribution of the fermion spins, which is only part of the total angular momentum. Fermions, being eigenstate of the projection $\vec{S}\vec{j}$, but with momentum $\vec{p}$, which is not parallel to $\vec{S}$, necessarily contribute to the total angular momentum also by some orbital part.

The relations (76) and (77) can be used also for the calculation of the higher momenta. Generally, if F is a function defined as

$$F(x) = \int K(p)\delta\left(\frac{p_0\nu + p_1\left|\vec{q}\right|}{M\nu} - x\right)d^3p,$$

then

$$\int x^n F(x)dx = \int\int K(p)x^n\delta\left(\frac{p_0\nu + p_1\left|\vec{q}\right|}{M\nu} - x\right)d^3p dx$$

$$= \int\int K(p)\left(\frac{p_0\nu + p_1\left|\vec{q}\right|}{M\nu}\right)^n\delta\left(\frac{p_0\nu + p_1\left|\vec{q}\right|}{M\nu} - x\right)d^3p dx$$

$$= \int K(p)\left(\frac{p_0\nu + p_1\left|\vec{q}\right|}{M\nu}\right)^n d^3p.$$

Application of this rule to Eqs. (76) and (77) gives after the substitution (51) and with the use of the second and third relation (46):

$$\int xg_2dx = -\frac{1}{6M}\int H(p_0)\left(p_0 - \frac{m^2}{p_0}\right)d^3p \tag{54}$$

$$\int x\left(g_1 + g_2\right)dx = \frac{1}{6M}\int H(p_0)\left(p_0 + 2m\right)d^3p \tag{55}$$

These equalities imply relation

$$\int x\left(g_1 + 2g_2\right)dx = \frac{1}{6M}\int H(p_0)\left(2m + \frac{m^2}{p_0}\right)d^3p, \tag{56}$$

which in the limit of the massless fermions coincides with the Efremov - Leader - Teryaev (ELT) sum rule [16]:

$$\int x\left(g_1 + 2g_2\right)dx = 0. \tag{57}$$

4 Discussion

In the previous sections we have studied the properties of the spin structure functions related to the system of quasifree fermions on mass shell. This system can be compared with the naive QPM, which is with embedded QCD corrections yet the basic tool for the analysis and interpretation of polarized and unpolarized deep inelastic scattering data. What is the difference between our approach and the naive QPM, if one speaks about the proton spin structure functions? To simplify this discussion, let us assume:

1) Spin contribution from the sea of quark-antiquark pairs and gluons can be neglected. Then three the fermions in our approach correspond to three the proton valence quarks. So, in this simplified scenario, the proton spin is generated only by the valence quarks.

2) In an accordance with the non-relativistic *SU(6)* approach the spin contribution of individual valence terms is given as

$$s_u = 4/3, \qquad s_d = -1/3. \tag{58}$$

Let us point out, in the given context the term valence quarks means nothing else, than three the fermions with defined momenta distribution, charge, mass and polarization.

Now in the standard naive QPM we have

$$g_1(x) = \frac{1}{2}\sum e_j^2 \Delta q_j(x) = \frac{1}{2}\left(\left(\frac{2}{3}\right)^2 \frac{4}{3} u_{val}(x) - \left(\frac{1}{3}\right)^2 \frac{1}{3} d_{val}(x)\right), \tag{59}$$

corresponding to two the quarks with distribution $u_{val}(x)$ and the one with distribution $d_{val}(x)$, which are normalized as

$$\int u_{val}(x)dx = \int d_{val}(x)dx = 1. \tag{60}$$

It follows, that

$$\Gamma_1 = \int g_1(x)dx = \frac{5}{18} \doteq 0.28. \tag{61}$$

This number overestimates more than twice the experimental value. Disagreement is generally interpreted as a contradiction with the assumption, that the proton spin is generated only by spins of the valence quarks.

Now let us calculate the Γ_1 in our approach. Let us denote momenta distributions of the valence quarks in the target rest frame by symbols $2h_u$ and h_d with the normalization

$$\int h_u(p_0)d^3p = \int h_d(p_0)d^3p = 1. \tag{62}$$

These distributions are connected with the $u_{val}(x)$ and $d_{val}(x)$ defined above by the relation

$$q_{val}(x) = \int h_q(p_0)\delta\left(\frac{p_0\nu + p_1 |\vec{q}|}{M\nu} - x\right) d^3p. \tag{63}$$

The charge weighted distribution (8) reads

$$H(p_0) = \sum e_j^2 \Delta h_j(p_0) = \left(\left(\frac{2}{3}\right)^2 \frac{4}{3} h_u(p_0) - \left(\frac{1}{3}\right)^2 \frac{1}{3} h_d(p_0)\right) \tag{64}$$

and one can easily check, that

$$\frac{1}{2}\int H(p_0)d^3p = \frac{5}{18}. \tag{65}$$

It follows, that the corresponding Γ_1 given by relation (53)

$$\Gamma_1 = \frac{1}{2}\int H(p_0)\left(\frac{1}{3}+\frac{2m}{3p_0}\right)d^3p$$

must satisfy

$$\frac{5}{18} \geq \Gamma_1 \geq \frac{5}{54}. \tag{66}$$

The maximum corresponds to the limit of static quarks and minimum to the limit of massless quarks.

Why these two very simple approaches for description of the target consisting of three the fermions differ regarding the prediction Γ_1? The reason is following. The standard formulation of the QPM is closely connected with the preferred reference system - infinite momentum frame (IMF). The basic relations between the distribution and structure functions like

$$g_1(x) = \frac{1}{2}\sum e_j^2 \Delta q_j(x), \qquad F_2(x) = x\sum e_i^2 q_i(x) \tag{67}$$

are derived with the use of approximation

$$p_\alpha = xP_\alpha, \tag{68}$$

which seems to be plausible in the IMF. Nevertheless, in the covariant formulation this relation is equivalent to the assumption, that the quarks are static with respect to the proton, since the velocities p_j/p_0 and P_j/P_0 are the same. In the proton rest frame it means $\vec{p}=0$. That is why both the approaches are equivalent for the static quarks but differ for the quarks, which have some internal motion inside the proton. In our approach we do not use assumption (68) and as a result if $p_\alpha \neq xP_\alpha$ we obtain different relations between the distribution and structure functions. In other words, the fact, that the experimental value Γ_1 is substantially under the value predicted by the naive QPM in standard formulation, can be in our approach interpreted as a direct consequence of the quark internal motion.

Next, let us mention another possible, but rather speculative effect of the quark internal motion. Until now we have assumed, that the momenta distributions G, H are spherically symmetric in the target rest frame. Due to this symmetry, the corresponding structure functions g_1 and g_2 do not depend on the variable $\vec{q}\vec{S} = |\vec{q}|\cos\omega$ (or qS, in covariant representation) despite the fact, that such terms are present in the starting integrals (70), (71) calculated in the Appendix. One can check, that a more general shape of the momenta distribution like

$$H\left(\frac{pP}{M},(pS)^2\right) = H\left(p_0,\left(\vec{p}\vec{S}\right)^2\right) \tag{69}$$

where the r.h.s. corresponds to the proton rest frame, could produce some $\vec{q}\vec{S}-$dependence of the functions g_1 and g_2. It follows in particular, that g_1, g_2 related to the longitudinally and transversally polarized targets could differ. Actually, in given approach, one could calculate also the unpolarized structure function F_2, as suggested in [13]. In the case of the disturbed spheric symmetry of the corresponding momenta distribution like in the formula (69), the F_2 in polarized experiment could depend on $\vec{q}\vec{S}$ as well.

Of course, the approach discussed above concerns the simplified scenario of the quasifree fermions on mass shell. Naive QPM represents only a first approximation for a description of real nucleon, but the consistent accounting for the quark internal motion as suggested in our approach can, in some aspects, improve this approximation considerably.

Nevertheless, in the realistic case of partons inside the nucleon the situation is still much more delicate. The interaction among the quarks and gluons is very strong, partons themselves are mostly in some shortly living virtual states, is it possible to speak about their mass at all? Strictly speaking probably not. The mass in the exact sense is well defined only for free particles, whereas the partons are never free. However one can assume the following. The relations obtained in the previous sections can be used as a good approximation even for the interacting quarks, but provided that the term *mass of quasifree parton* is substituted by the term *parton effective mass*. By this term we mean the mass, which a free parton would have to have to interact with the probing photon equally as the real, bounded one. Intuitively, this mass should correlate to Q^2: a lower Q^2 roughly means, that the photon "sees" the quark surrounded by some cloud of gluons and quark-antiquark pairs as a one particle - by which this photon is absorbed. And on contrary, the higher Q^2 should mediate interaction with more "isolated" quark. Moreover, one should accept that the value of the effective mass can even for a fixed Q^2 fluctuate. Such phenomenological model was suggested in [13], but unfortunately calculation was based on the form of quark polarization vector which is not correct. Despite of that, the general considerations in mentioned paper can be sensible. Corresponding numeric recalculation with the correct input obtained in the present study for the invariants A, B, C, D [relations (29),(43)] should be done in a separate paper.

5 Summary and conclusion

In the present paper we have studied the spin structure functions of the system of quasifree fermions on mass shell. The main results can be summarized as follows:

1) Using consistently covariant description of this simple system, we have shown how the structure functions depend on the internal motion of the fermions. In particular, we have shown, that the moment Γ_1 reaches the maximal value Γ_{max} for the static fermions ($p_0 = m$) and minimal value $\Gamma_{min} = \Gamma_{max}/3$ in the limit of massless fermions ($m \ll p_0$).

2) We have shown, what summation rules follow from the obtained spin

structure functions. Further we have shown, how these rules are related to some summation rules well known from the QPM phenomenology.

3) We have done a comparison with the corresponding relations for the structure functions following from the standard formulation of the naive QPM. Both the approaches are basically equivalent for the static quarks. Differences for quarks with internal motion inside the proton are result of the conflict with the assumption $p_\alpha = xP_\alpha$, which is crucial for derivation of the relations between structure and distribution functions in the standard QPM.

4) The difference between the experimental value Γ_1 for the proton and the corresponding value expected from the naive QPM, or at least a part of this difference, can be interpreted as a consequence of the quark motion inside the proton.

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A Calculation of the integrals related to G_P, G_S

Integrals in the relations (44), (45) expressed in the target rest frame read

$$G_P = -\frac{m}{2M^2\nu}\int H(p_0) \tag{70}$$

$$\times\frac{\vec{p}\vec{S}}{p_0+m}\left[1+\frac{1}{m}\left(p_0-\frac{\vec{p}\vec{q}-\left(\vec{p}\vec{S}\right)|\vec{q}|\cos\omega}{|\vec{q}|^2\sin^2\omega}\nu\right)\right]\delta\left(\frac{pq}{M\nu}-x\right)\frac{d^3p}{p_0},$$

$$G_S = \frac{m}{2M\nu}\int H(p_0) \tag{71}$$

$$\times\left[1+\frac{\vec{p}\vec{S}}{p_0+m}\frac{1}{m}\left(\vec{p}\vec{S}-\frac{\vec{p}\vec{q}-\left(\vec{p}\vec{S}\right)|\vec{q}|\cos\omega}{|\vec{q}|^2\sin^2\omega}|\vec{q}|\cos\omega\right)\right]\delta\left(\frac{pq}{M\nu}-x\right)\frac{d^3p}{p_0},$$

where $\cos\omega \equiv \vec{q}\vec{S}/|\vec{q}|$. For integration we use the orthonormal system in which

$$\vec{p} = p_1\vec{e}_1+p_2\vec{e}_2+p_3\vec{e}_3, \quad \vec{e}_1=-\frac{\vec{q}}{|\vec{q}|}, \quad \vec{e}_2=\frac{\vec{S}-(\vec{S}\vec{e}_1)\vec{e}_1}{\sqrt{1-(\vec{S}\vec{e}_1)^2}}, \quad \vec{e}_3=\vec{e}_1\times\vec{e}_2, \tag{72}$$

so one gets

$$\vec{p}\vec{q} = -p_1 \left|\vec{q}\right|, \qquad \vec{p}\vec{S} = -p_1 \cos\omega + p_2 \sin\omega, \qquad \cos\omega \equiv \frac{\vec{q}\vec{S}}{\left|\vec{q}\right|}. \tag{73}$$

After the substitution $p_2 = p_T \cos\varphi,\ p_3 = p_T \sin\varphi$ and taking into account that the terms proportional to $\cos\varphi$ disappear, the integrals can be rewritten

$$G_P = \frac{\cos\omega}{2M^2\nu}\int H(p_0)\left(p_1 + \frac{\nu}{\left|\vec{q}\right|}\frac{p_1^2 - p_T^2\cos^2\varphi}{p_0 + m}\right) \tag{74}$$

$$\times\delta\left(\frac{p_0\nu + p_1\left|\vec{q}\right|}{M\nu} - x\right)\frac{p_T dp_1 dp_T d\varphi}{p_0},$$

$$G_S = \frac{m}{2M\nu}\int H(p_0)\left(1 + \frac{p_T^2\cos^2\varphi}{m\,(p_0 + m)}\right)\delta\left(\frac{p_0\nu + p_1\left|\vec{q}\right|}{M\nu} - x\right)\frac{p_T dp_1 dp_T d\varphi}{p_0}, \tag{75}$$

where $p_0 = \sqrt{m^2 + p_T^2 + p_1^2}$. Integration over φ gives

$$G_P = \frac{\pi\cos\omega}{M^2\nu}\int H(p_0)\left(p_1 + \frac{\nu}{\left|\vec{q}\right|}\frac{p_1^2 - p_T^2/2}{p_0 + m}\right)\delta\left(\frac{p_0\nu + p_1\left|\vec{q}\right|}{M\nu} - x\right)\frac{p_T dp_1 dp_T}{p_0}, \tag{76}$$

$$G_S = \frac{\pi m}{M\nu}\int H(p_0)\left(1 + \frac{p_T^2/2}{m\,(p_0 + m)}\right)\delta\left(\frac{p_0\nu + p_1\left|\vec{q}\right|}{M\nu} - x\right)\frac{p_T dp_1 dp_T}{p_0}, \tag{77}$$

Further, using the relation

$$\frac{\left|\vec{q}\right|}{\nu} = \sqrt{1 + 4M^2x^2/Q^2} \tag{78}$$

one can check, that the argument of $\delta-$ function equals zero for

$$p_1 = \mathbf{p_1} \equiv \frac{Mx - m_T^2/Mx}{\sqrt{1 + 4m_T^2/Q^2} + \sqrt{1 + 4M^2x^2/Q^2}}, \qquad m_T^2 \equiv m^2 + p_T^2. \tag{79}$$

This is the first root of the corresponding quadratic equation, the second one is excluded, since in the effect of the $\delta-$ function this root is compatible only with negative energy p_0. The energy corresponding to the root (79) is

$$p_0 = \mathbf{p_0} \equiv Mx - \frac{\mathbf{p_1}\left|\vec{q}\right|}{\nu} = Mx - \mathbf{p_1}\sqrt{1 + 4M^2x^2/Q^2}. \tag{80}$$

Then in an accordance with the rule

$$\delta(f(x))dx = \sum_j \frac{\delta(x - x_j)}{\left|f'(x_j)\right|}dx, \qquad f(x_j) = 0 \tag{81}$$

the $\delta-$ function in the integrals can be rewritten

$$\delta\left(\frac{p_0\nu + p_1\,|\vec{q}|}{M\nu} - x\right) dp_1 = \frac{M\delta(p_1 - \mathbf{p_1})dp_1}{\mathbf{p_1}/\mathbf{p_0} + \sqrt{1+4M^2x^2/Q^2}} \tag{82}$$

and afterwards the integrals are simplified

$$G_P = \frac{\pi\cos\omega}{M\nu}\int_0^{p_{T\,\max}} H(\mathbf{p_0})\left(\mathbf{p_1} + \frac{\nu}{|\vec{q}|}\frac{\mathbf{p_1^2} - p_T^2/2}{\mathbf{p_0}+m}\right)\frac{p_T dp_T}{\mathbf{p_1} + \mathbf{p_0}\sqrt{1+4M^2x^2/Q^2}}, \tag{83}$$

$$G_S = \frac{\pi m}{\nu}\int_0^{p_{T\,\max}} H(\mathbf{p_0})\left(1 + \frac{p_T^2/2}{m\,(\mathbf{p_0}+m)}\right)\frac{p_T dp_T}{\mathbf{p_1} + \mathbf{p_0}\sqrt{1+4M^2x^2/Q^2}}, \tag{84}$$

where $\mathbf{p_1}$ and $\mathbf{p_0}$ depend on p_T according to Eqs. (79) and (80). For the numeric calculation one should know the upper limit $p_{T\,\max}$ for given x, Q^2 and $\mathbf{p}_{0\,\max}$. After inserting $\mathbf{p_1}$ from Eq. (79) into Eq. (80) one gets equation for m_T^2

$$\frac{\mathbf{p}_{0\,\max} - Mx}{\sqrt{1+4M^2x^2/Q^2}} = -\frac{Mx - m_T^2/Mx}{\sqrt{1+4m_T^2/Q^2} + \sqrt{1+4M^2x^2/Q^2}} \tag{85}$$

Instead of m_T^2 it is useful to solve this equation first for $y = \sqrt{1+4m_T^2/Q^2}$ obtaining the two roots

$$y_\pm = \frac{A \pm \sqrt{A^2 + 4a(\mathbf{p}_{0\,\max} + a)}}{2a}, \qquad A \equiv \frac{\mathbf{p}_{0\,\max} - Mx}{\sqrt{1+4M^2x^2/Q^2}}, \qquad a \equiv \frac{Q^2}{4Mx}. \tag{86}$$

Since $y_- < 0$, this root is excluded. The second root y_+ after some computation implies

$$m_{T\,\max}^2 = Mx(2\mathbf{p}_{0\,\max} - Mx) + \frac{(\mathbf{p}_{0\,\max} - Mx)^2}{1 + Q^2/4M^2x^2}, \qquad p_{T\,\max} = \sqrt{m_{T\,\max}^2 - m^2}. \tag{87}$$

In this way we have the recipe how to calculate the integrals related to the structure functions G_P, G_S corresponding to the distribution $H(p_0)d^3p$.

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